PHY250/251 – Quantum Mechanics

Problem class set 3

Dr. Matthew Malek

The potential for the one-dimensional linear harmonic oscillator is: $V(x) = \frac{1}{2}m\omega^2 x^2$. Thus, the Hamiltonian for the quantum harmonic oscillator (Q.H.O.) is:

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2 x^2$$

1. Consider the following pair of operators:

$$\hat{A} = \sqrt{\frac{m\omega}{2\hbar}}(\hat{x} + \frac{i}{m\omega}\hat{p})$$

$$\hat{A}^\dagger = \sqrt{\frac{m\omega}{2\hbar}}(\hat{x} - \frac{i}{m\omega}\hat{p})$$

(a) Evaluate the commutator $[\hat{A}, \hat{A}^\dagger]$

(b) Show that you can express the Hamiltonian $\hat{H}$ in terms of $\hat{A}^\dagger$ and $\hat{A}$.

(c) Evaluate the commutator $[\hat{H}, \hat{A}^\dagger]$.

(d) Evaluate $\hat{H}\hat{A}^\dagger\psi_n(x)$ where $\psi_n(x)$ is an eigenstate solution of the 1D Q.H.O.